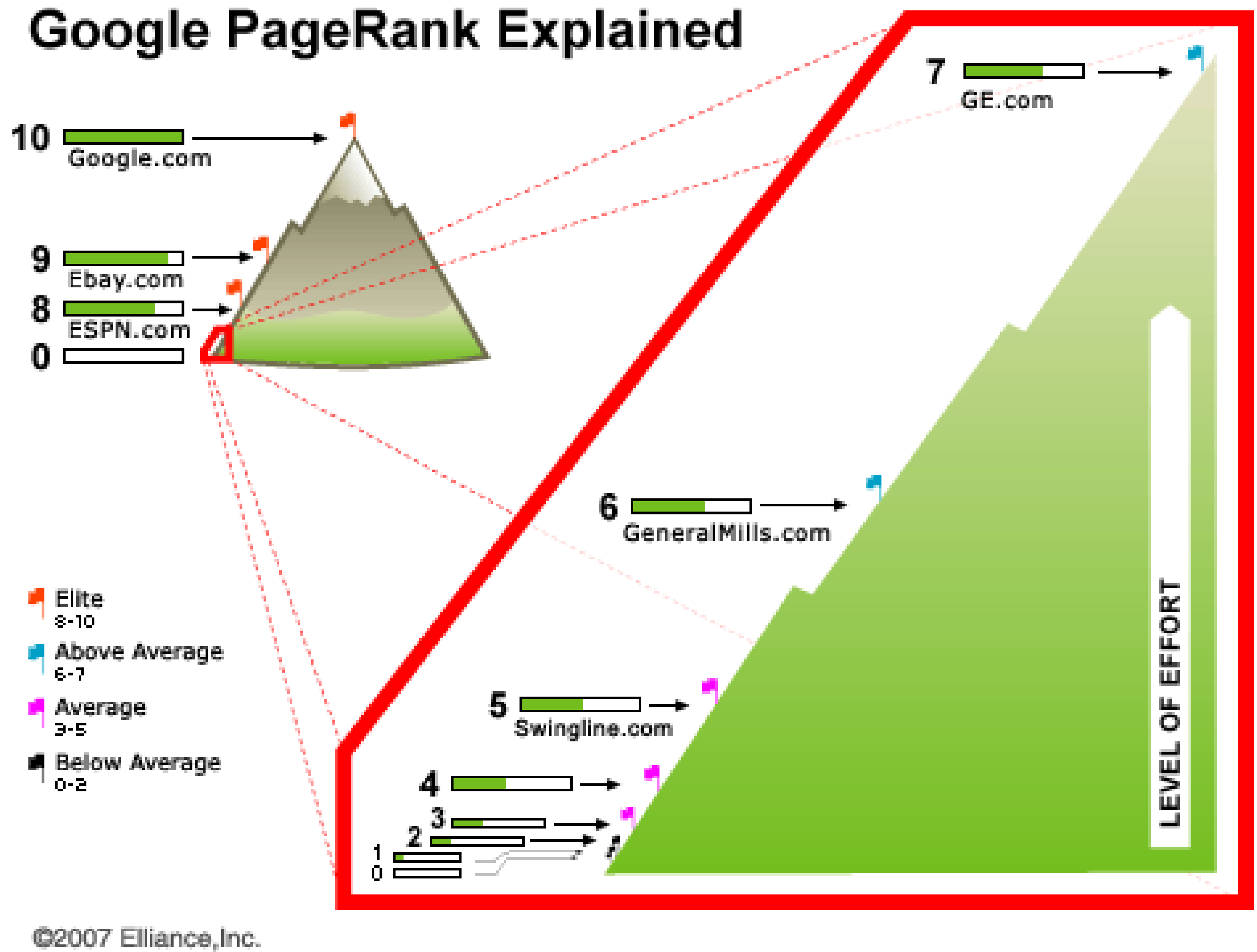
# PageRank

Hung-yi Lee

# Search


http://incomebully.com/does-pr-pagerank-still-matter/




http://www.hobo-web.co.uk/google-pr-update/

# PageRank

- Information of 2008

**Rank：8**

Google台灣首頁：www.google.com.tw
Youtube台灣首頁：http://tw.youtube.com
台灣大學網站首頁：www.ntu.edu.tw

**Rank：7**

痞客邦首頁：www.pixnet.net
104人力銀行：www.104.com.tw
無名小站首頁：www.wretch.cc

**Rank：6**

博客來網站：www.books.com.tw
聯合新聞網首頁：http://udn.com
天下雜誌網站首頁：www.cw.com.tw

**Rank：5**

推推王網站：http://funp.com
愛情公寓網站：www.i-part.com.tw
KKman網站首頁：
www.kkman.com.tw

**Rank：4**

工頭堅部落格：http://worker.bluecircus.net
白木怡言部落格：www.yubou.tw
RO仙境傳說網站：http://ro.gameflier.com

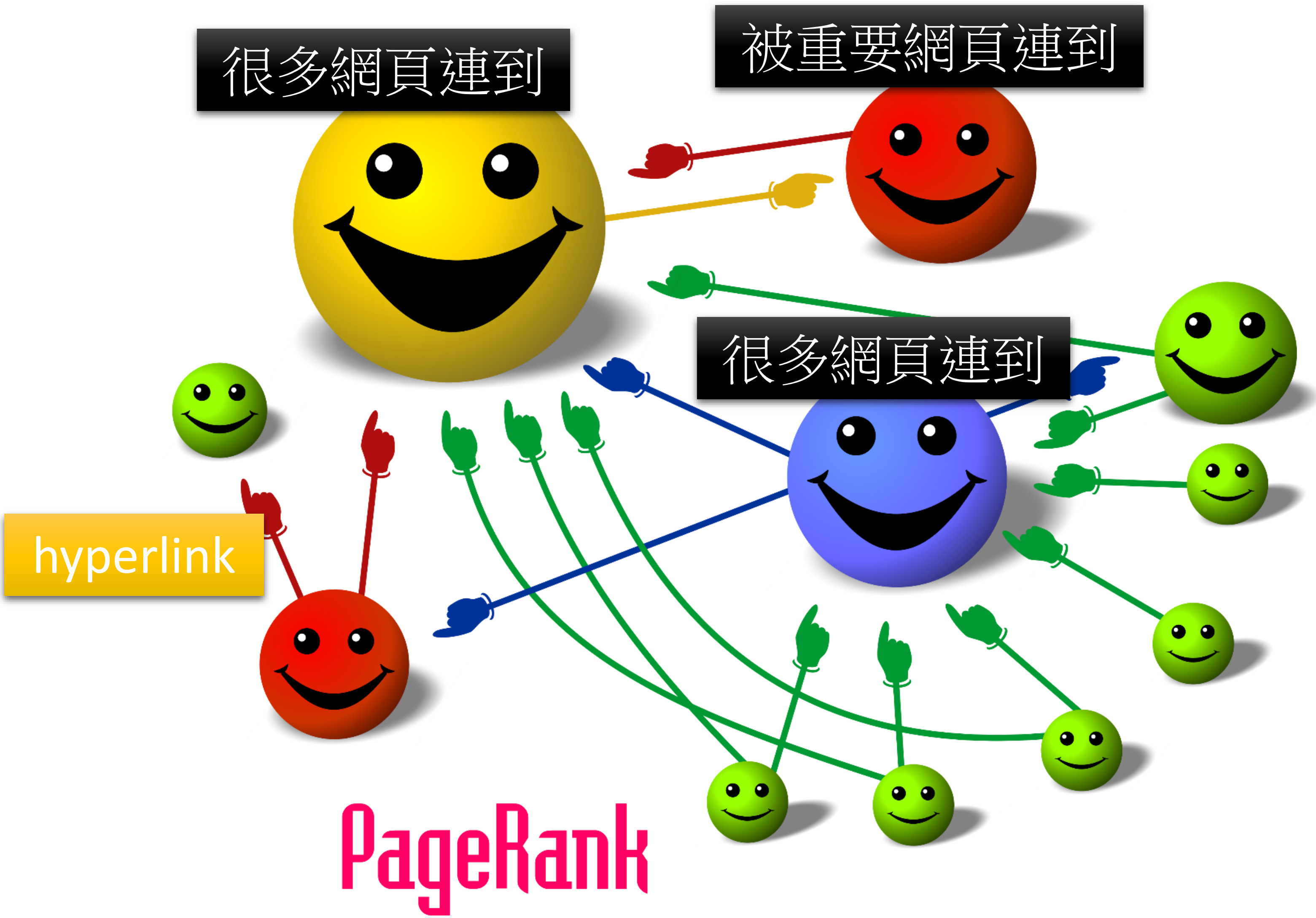
Source: http://terrylogin.blogspot.com/2008/09/pagerank.html

# PageRank

# PageRank

- *Webpages with a higher PageRank are more likely to appear at the top of Google search results.*
- *PageRank relies on the uniquely democratic nature of the web by using its vast link structure as an indicator of an individual page's value.*
- ***Google interprets a link from page A to page B as a vote, by page A, for page B**.*

# Importance



# PageRank

## The PageRank Citation Ranking: Bringing Order to the Web

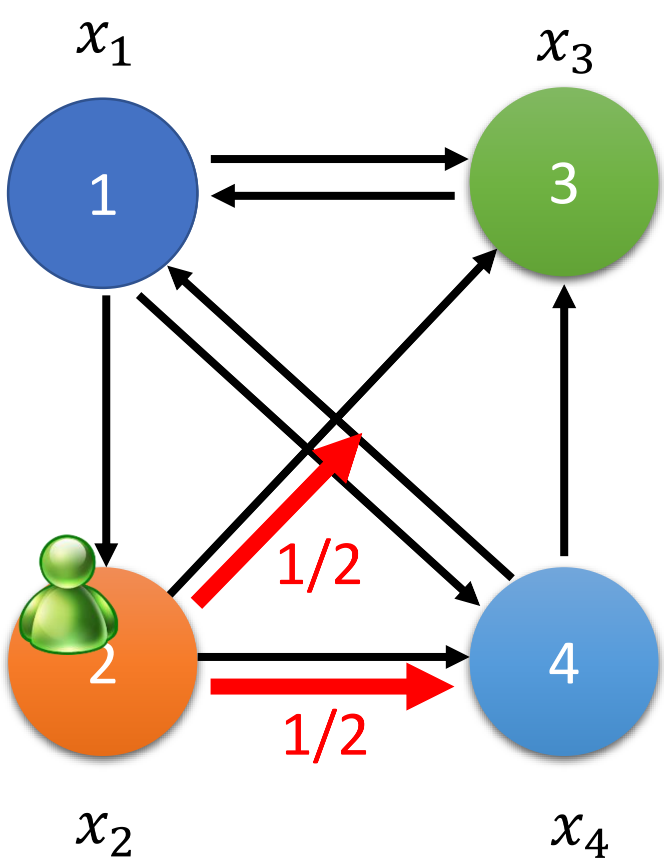
January 29, 1998

**Abstract**

The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively about the relative importance of Web pages. This paper describes PageRank, a method for rating Web pages objectively and mechanically, effectively measuring the human interest and attention devoted to them.

We compare PageRank to an idealized random Web surfer. We show how to efficiently compute PageRank for large numbers of pages. And, we show how to apply PageRank to search and to user navigation.

# Importance - Formulas




$$x_1 = x_3 + \frac{1}{2}x_4$$

$$x_2 = \frac{1}{3}x_1$$

$$x_3 = \frac{1}{3}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_4$$

$$x_4 = \frac{1}{3}x_1 + \frac{1}{2}x_2$$

Consider a random surfer

# Importance - Formulas

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$Ax = x$$

$$x_1 = x_3 + \frac{1}{2}x_4$$

$$x_2 = \frac{1}{3}x_1$$

$$x_3 = \frac{1}{3}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_4$$

$$x_4 = \frac{1}{3}x_1 + \frac{1}{2}x_2$$

The solution x is in the eigenspace of eigenvalue $\lambda = 1$

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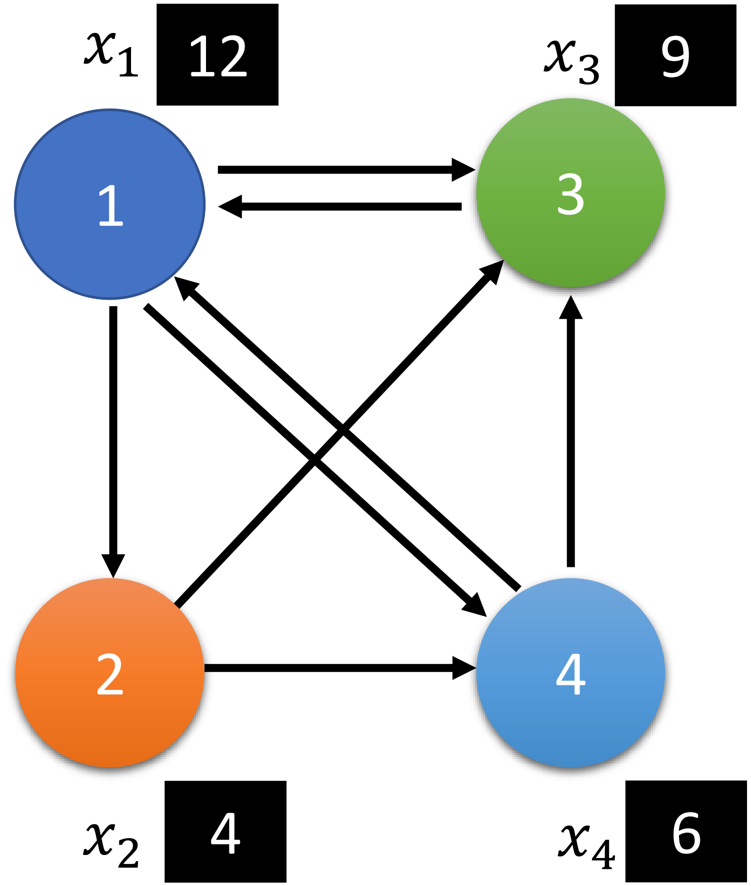
$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

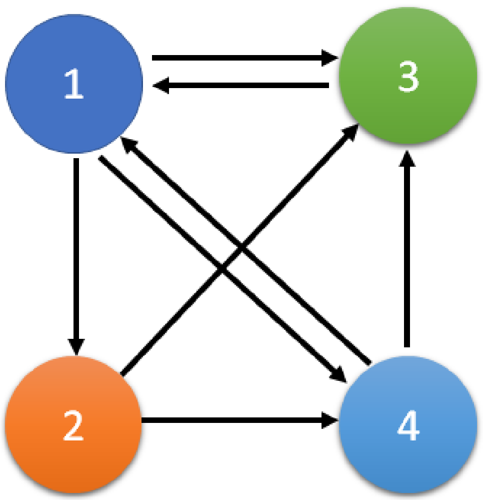



$$Ax = x$$

The solution x is in the eigenspace of eigenvalue $\lambda = 1$

$$Span\{[12 \quad 4 \quad 9 \quad 6]^T\}$$

# Eigenvalue = 1




$$Ax = x$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

Column-stochastic Matrix

Column-stochastic matrix always have eigenvalue $\lambda = 1$ Proof

How about the Dangling nodes (只入不出)?

# Unique Ranking?

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

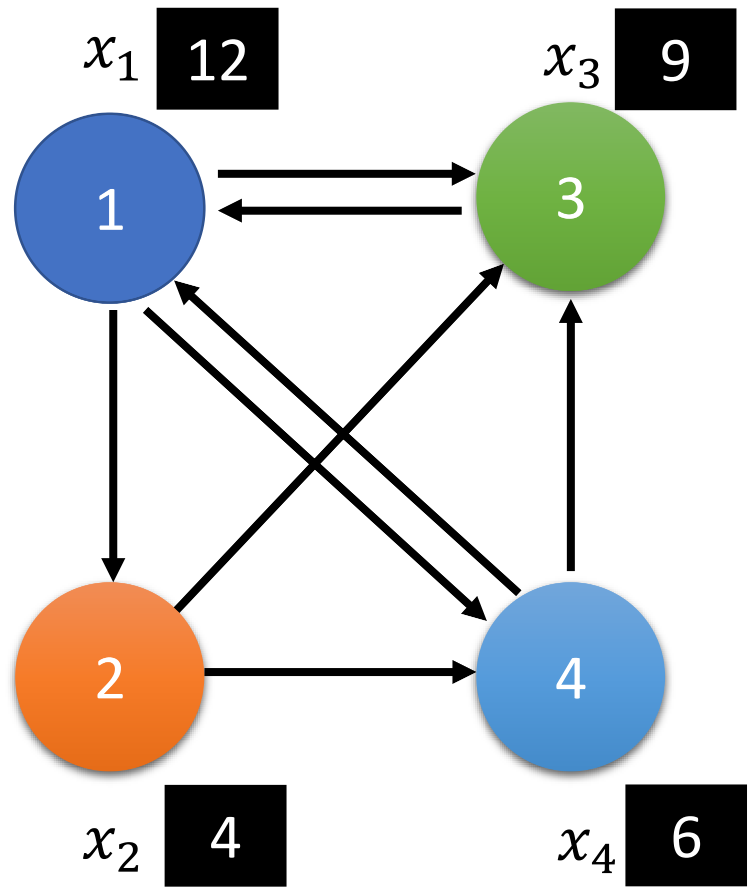
Having eigenvalue $\lambda = 1$

The dimension of the subspace is must 1

constraint

Unique Ranking

Unique Score



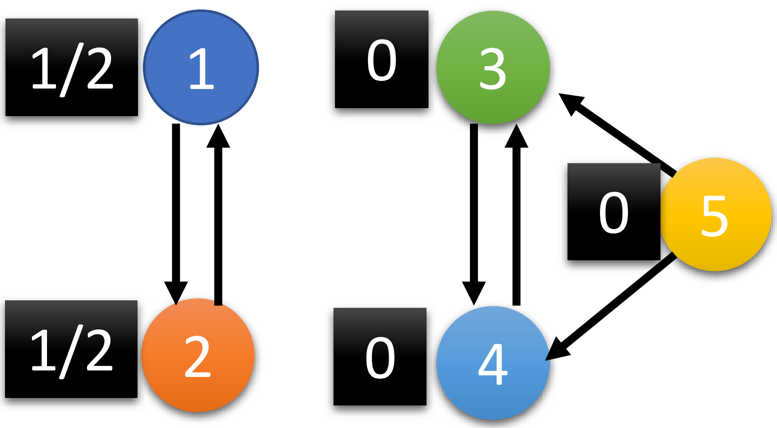
# Unique Ranking?

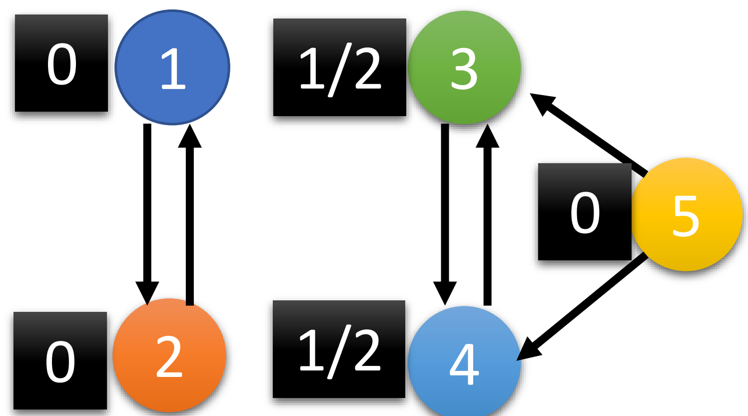
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

How about dimension > 1

Dim for $\lambda = 1$ is 2

Basis:




Any linear combination is in the eigenspace

Not Unique Ranking

# Unique Ranking?

Can it be non-uniform?

All entries are 1/n

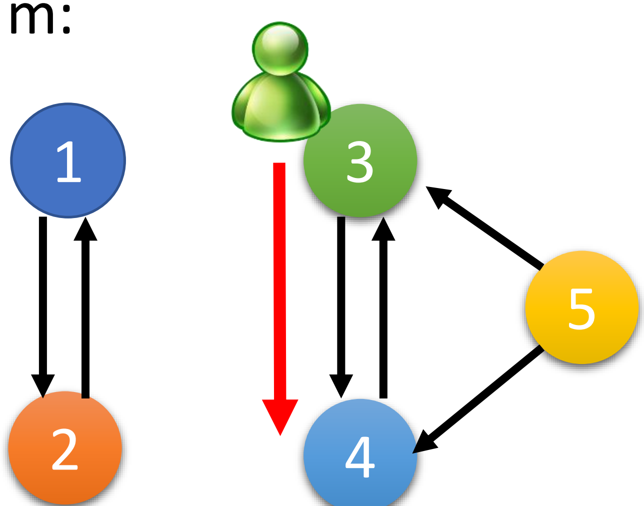
$$\mathbf{M} = (1-\overset{\color{red}{0.15}}{m})\mathbf{A} + m\mathbf{S}$$

Follow the link

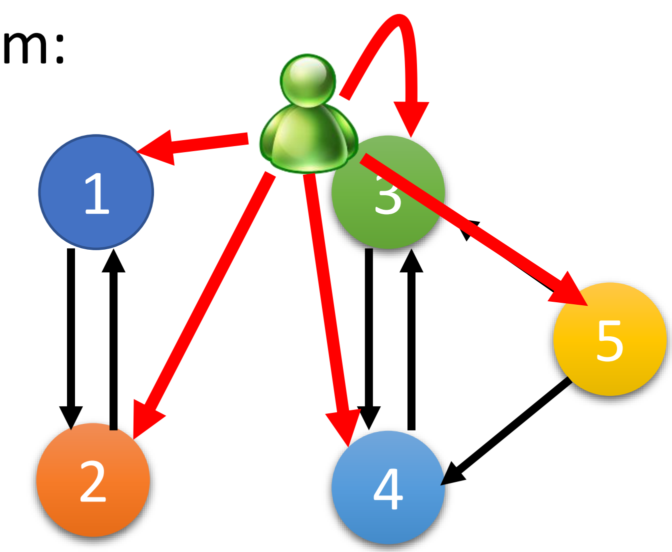
random

There are two ways to surf the web

Prob 1 – m:




Prob m:



# Unique Ranking?

$$\mathbf{M} = (1 - m)\mathbf{A} + m\mathbf{S}$$

- Unique ranking
- For M, the dim of the eigenvalue $\lambda = 1$ is 1

M is Column-stochastic matrix and "positive"

Proof

Dim = 1

Hint: For M, the eigenvectors for eigenvalue $\lambda = 1$ are all "positive" or "negative"

# Power method

M is very large

Find $x^*$, such that $x^* = Mx^*$, $\|x^*\|_1 = 1$

Start from $x_0$, $\|x_0\|_1 = 1$

$$x_1 = Mx_0$$

$$x_2 = Mx_1$$

$$\vdots$$

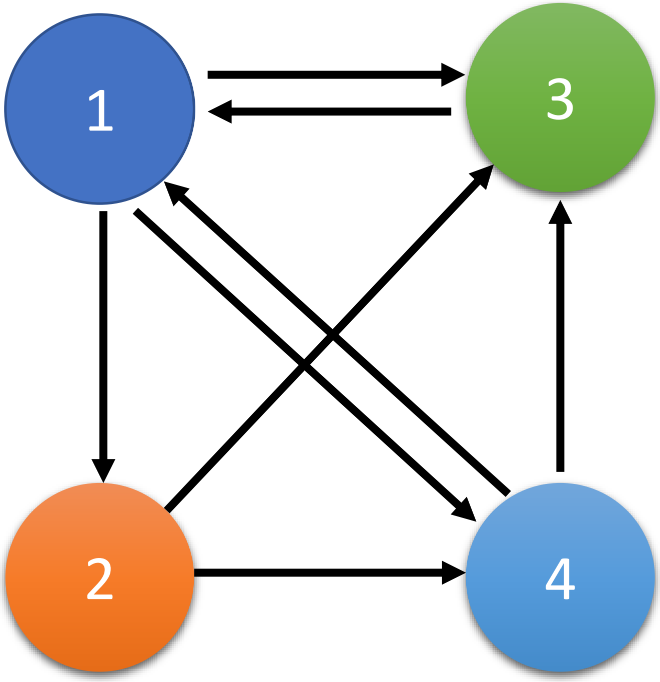
$$x_k = Mx_{k-1}$$

If $k \to \infty$

$$x_k = x^*$$

Proof

# Power method - Example




$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

# Actually ……

- **The Last Toolbar Pagerank Update was December 2013**
- Google declared thereafter: "P*ageRank is something that we haven't updated for over a year now, and we're probably not going to be updating it again going forward, at least the Toolbar version.*"